

Unmissing the Missing Matter in Neo-Kaluza-Klein Universes*

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Abstract

Uncompactified KK universes are so intrinsically connected to the otherwise only empirically required “missing” Dark Matter (DM), that:

- 1) They yield a simple prediction which explains both the enigma of the extra-dimensions’ (XD) unobservability and the enigma of the present DM. The two enigmas are “annihilated” into the hypothesis of “missing light”, or better of “photonland”. This eliminates the very need to hypothesize/search/find DM-candidates of exceptional/exotic properties to explain their darkness.
- 2) An early, spontaneous gravitational XD-collapse of their natural 5D-DM replaces KK’s compactification mechanisms and cylindricity condition, and (partly?) eliminates the quandary of the radically insufficient density fluctuations.

(I) Recently there has been renewed interest and progress in the KK-5D-cosmology. In many of the new variations:

- A) Artificial, ad hoc geometrical preconditions are eliminated, to be eventually replaced by physical processes. Thus, unless otherwise proved
- B) The present universe – not merely its primordial “ape” – is occupied by 5D-bodies: KK-solitons (5D blackhole analogues) [1] and/or others [e.g. 2], if not even ($n > 5$) D-densities.

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Since these two changes enable fundamental advancements and research directions, we label these universes “neo-KKU”. Among other things, one can see that:

1) 4D-GR is **embedded** in a flat 5D-space [3], thus curvature can also be seen as exterior.

2) There exists an equational equivalence between empty 5D-GR and 4D-GR with sources [4, 5, 6]. This stresses the necessity to decipher **how** could our universe have appeared from pure geometry [e.g. 7] (Einstein’s famous vision). In the old-KK paradigm geometrical conditions helped the vision to resemble a miracle. Neo-KK workers have invested much efforts [e.g. 5,8,9] in order to uncover the **physical** processes behind this “induced matter” interpretation. However, many still hope this “induced” matter to be “a manifestation of pure geometry” [5].

3) As the mathematics of “inducing” 4D-GR plus sources from empty 5D-GR was generalized [10, 11, 12] to n D, the neo-KK paradigm automatically includes universes which generate/allow dimensional collapses from $(n > 5)$ D; and into $(n < 4)$ D densities/fluctuations/particles. This allows condition B above, and our hypothesis below, to assume non-zero densities of $n > 5$.

(II) Independent of the KK paradigm, cosmological-theoretical **and** astronomical-observational discrepancies almost compel us to theorize and find DM-candidates:

a) Calculations of galaxy/cluster sizes definitely require a prebaryonic collapse (but see more below). Since the source of such a collapse is an enigma, theories were artificially/ad-hocly stretched to hypothesize the existence of prebaryonic matter.

b) The presently known matter makes up only a small percentage of the gravitation needed **today** to prevent the galaxies from escaping their clusters, and from losing their own peripheries.

c) It was recently shown that “any attempt to explain the mass discrepancy... using alternative gravity instead of DM comes at the price of having to abandon the equivalence principle. DM can now be seen in a new light: it is indispensable...” [13].

Attempting to replace the ad-hocly tailored geometrical transformations from 5D to 4D-GR, by physical processes by which 5D-densities collapse/confine (see below) to 4D, we can no longer appriorily expect them to be elegantly perfect and total. As condition A generates condition B – unless proved/assumed to have been completely collapsed into 4D-densities — it is **essential** to analyze and conceive our present universe five-dimensionally. This is of great

importance in two major respects:

- 1) In old-KKU the primordial 5D-densities are almost¹ untestable: geometrical preconditions ad-hocly **equate** them to GR's 4D-densities, which denies us differential-hence-testable-predictions. By contrast, in neo-KKU the possibility (shown to be a necessity) of great **present** abundance of 5D-densities greatly differs from any conventional prediction, and is shown to be testable.
- 2) These present-universe-5D-densities are shown to be **natural** DM-candidates; are extensively investigated as solitons [e.g. 1, 7, 14, 15] and other forms of "extended objects"; are proved below to yield more than DM-candidates.

The neo-KK's development is still partial. Firstly, only **indications** for such physical processes were found. Secondly, there still exist reservations/questions/unclearities concerning our validity in analyzing physics 5-dimensionally [3, 5, 15, 16] (but see below). Thirdly — without entering this complex issue here — the appearance of our physical universe from **absolutely**² empty 5D-GR might remain a mathematical "miraculous³ creation" of 4D- "something" from 5D- "nothing".

However, one thing is clear: if a 5D-universe (5DU) does not merely "induce" or "manifest as" [5], but physically **produce** our 4DU — these collapse processes (see below) also take time. If they started before our 4DU has emerged, neo-KKU intrinsically predicts extra prebaryonic **time**. This prediction neither contradicts nor necessitates extra prebaryonic **matter**. Therefore, unless compelled by a – c above, some versions of neo-KKU could predict **no** prebaryonic missing matter.⁴ Secondly, as the prebaryonic collapse — **and** its smaller galaxy horizons [4] — are (presently) integral parts of neo-KKU, they seem to break down if the prebaryonic collapse is refuted. Surprisingly, however, a several hundred thousands year long gravitational XD-collapse is predicted below. If verified, there **might** be no prebaryonic matter **and** no prebaryonic collapse — only prebaryonic time, in which only inflationary expansion takes place, **without** "mechanisms ... to pump entropy into the 4DU" [5].

¹Compactified 5D-KK radically (10^{22}) disagree [5] with observations, but see below.

²In [17] we try to extend it to 5D **fluctuations** of (\pm) curvatures/densities **around** zero.

³"The fact that... the sourceless field equations lead to ... **with** source matter, constitutes the central miracle of Kaluza-Klein theory." [5]

⁴Even more so in time-dependent-G-models, and other models which allow/produce extended inflations.

On the other hand, the **present** universe seems to contain vast amounts of DM. While several approaches exist in accepting/rejecting various candidates, **everybody** (still) believes that their unobservability is due to some special or even exotic properties. This has driven us to theorize DM-candidates which are not intrinsically predicted (and sometimes even not tolerated) by physical theories, unless stretched/burdened by extra- theoretical matters. . . In sharp contrast, in neo-KKU the primordial matter needs only remain in its natural 5D-density to form the present-epoch-DM! So, as we are compelled to predict both the prebaryonic collapse and the great present abundance of 5D extra matter, just a simple variation is needed: The prebaryonic collapse from 5D to 4D (or from $n \geq 5$ to $n \leq 4$, but see below) was **partial** — all the 5D density was condensed, yet only a small part of it fully collapsed to (or arrested in, below) 4D.

A partial collapse might at first look strange. Yet, “who ordered” a perfect “production line” which does not cease before **all** its 5D “raw material” became fully collapsed to **our** 4D “specification”?! (see also [3, 6]). Only the inflationary expansion/XD-collapse processes dictate the collapse ratio of 4D-matter/5D-DM. So, by contrast to old-KKU, **all** neo-KKU must obey (and eventually predict) the DM/matter ratio! In addition, this also upgrades the paradigm’s refutability: Any deviation from empirical constraints on the present DM/matter ratio **refutes** the neo-KK paradigm. While these constraints are still vague, satellites new observations are expected to rapidly clarify them.

Predicting the present DM-ratio – instead of only explaining the DM phenomena – will upgrade the KK-paradigm to eventually become fully testable. But prior to this, we must realize that our predicted, natural **neo-KK’s nD-DM are presently bugged by two main drawbacks:**

- 1) Explaining/predicting the missing-dark-matter by missing-dark-dimensions does not contribute too much conceptually — unless the XD-unobservability itself is predicted. Yet, old-KKU rely on ad-hoc explanations of it. E.g. [5] “They are a priori cylindrical. No mechanism is suggested to” (sufficiently) “explain why physics depends on the first four coordinates, but not on the extra ones”.
- 2) Hypothesizing a natural 5D-DM, we ascribe “5D feathers” not only to the primordial universe, but to our present 4DU. This is a bizarre prediction; much more than the elegantly structured solitons show. If such “extended-feathers” have some 90% of the total messenergy, our 4DU resembles a small

4D-body of a vast, surrealistic peacock. But beyond being bizarre, this prediction is dangerous. As 5D-densities vary by $1/R^4$ [1] — and nD-densities by $1/R^n$! — nD “unobservable peacocks” might be of monstrous messenergies, no less than “our regular” 4D-blackholes. Therefore, by “just” proposing $n > 5$, these elegantly hidden nD-blackhole-analogues could have “saved” us even if we had to explain 99.99% of DM!

Reflecting only the **assumed** XD-unobservability, and dangerous in higher D models, KK’s nD-DM cannot be trusted before we are able to **predict** the XD-unobservability; and **here our main hypothesis begins**.

(III) The present work proposes a more essential connection between XD and DM. It deciphers and eliminates the enigmas of both the DM and the unobservability of XD(s). It is tested against the CMBR’s photon density, and drastically reduces the amazing present insufficiency of the primordial density-fluctuations. And above all, it clarifies that it is the behavior of our friends the photons – not of new/exotic candidates – which restricts our observations to 4DU! Thus all XD-matter (XDM) ”becomes” (simply remains) totally unobservable.

As 5D-solitons/other natural nD-DM are (trivially) predicted and no artificial pre-condition ”irons” the XD, their width in XD might be large or small, but not negligible as in the old KKKU. This (alone) yields a simple fundamental possibility hence hypothesis: If **photons**⁵ are intrinsically **unable** to propagate in XD (s), a great fraction of any neo -KKU matter cannot exchange photons with us, which makes us **XD-blind!** Since XDM does not even absorb photons, it cannot distort our bolometrical readings, except by lensing. Thus, XDM is not just dark (like baryonic DM-candidates) but bolometrically **traceless**.

While in old-KKKU gravitation in XD (artificially) loses its significance, in neo-KK it does not. By contrast (to gravitons ?!), photons are here hypothesized – and (almost) empirically proved below – to have zero XD-velocity. If XDM cannot exchange photons with us, but does interact gravitationally with us, the (great) discrepancy which dictates the ”missing matter” is predicted!

Our hypothesis is in essential harmony with Wesson & Liu’s hypothesis [18] (below) that massive particles’ electric charge is (a function of) their XD- momentum. It is however in clear contradiction with Gogberashvili’s

⁵But not the massive **virtual** photons.

new 5D-theory [19] which seems to rely on photons' XD-velocity. As the neo-KKU are quite testable [e.g. 4,7,14,20-22] much of this disagreement can be resolved empirically (see below).

To prove our hypothesis we shall use the radiation and matter-dominated-eras as our lab. If photons had velocity in XD, their (strong) radiation-pressure in the radiation dominated era would have hyper-scattered the entire fireball in the 5D hyperspace⁶. As its density would then diminish by a higher **power**⁷ of their radius, the fireball's (radiation + matter) densities and temperatures are definitely predicted to drop much faster than by any standard cosmology. This drastically shortens "our" 300,000 years' radiation dominated period, and recombination takes place. As photons decouple from matter, their expansion is no longer decelerated by the matter densities. This transparency is crucial to our hypothesis: it turns the photonball absolutely free to hyperexpand in nD. So, **if** photons **had** XD-velocity, they would have been at least 5D-hyperscattered, decreasing their density and temperature significantly below the standard predictions. This would definitely contradict the (already successfully predicted) observed photon's number-density and temperature in the CMBR!⁸

Since the above cosmological eras are not "ideal labs", it is unclear whether their radiation's and matter's expansions in XD's are cut off by the photons'/field's own properties or by some external forces. We do not attempt (here) to specify the first, but to refute the second. At the radiation dominated era the prebaryonic collapse processes are definitely over, thus leave the photons free to expand in XD. But, one may cautiously suspect the yet insufficiently known inflationary XD-collapse (but see below) to have extended into the radiation dominated era. However, any such pressure is related to/ represented by a non-zero cosmological constant. At this era – and certainly at the far later transparent era which is crucial here – the cosmological constant vanishes, hence nothing counterforces the radiation pressure. While several possible Λ -calculations exist – it is safe to assume that at this 300,000 year long era the $\Lambda \sim 0$ related field(s) become weaker than the

⁶Although one may assume the XD to be smaller than the relevant mean free paths, this in itself cannot prevent any radiation-matter interaction.

⁷See for example 5D-solitons' density $\propto 1/R^4$ [1].

⁸Beyond changing the particles mean free paths values and relations to the altered radia of the universe, thereby disrupting (quite) established and accepted cosmological phases, hence also the universe's age.

matter fields, which are themselves weaker than the radiation pressure. To quote from a recent 10D-KK example, “At the end of the inflationary phase transition, a radiation dominated era is brought about by demanding the total effective cosmological constant to vanish” [23].

With (almost?) no external pressures in XD, only the field’s own properties can cut off the photons’ XD-velocity! So, our best friends the photons are self-confined to 4DU. **Our** photon-dependent **observations** are therefore encapsulated in “photonland”! “Luckily”, photonland is not flatland, but overlaps our 4DU.

If photons were merely oppositely charged pairs, Wesson & Liu’s hypothesis would suffice to predict their annihilations to cancel their XD-momenta, hence XD-velocities (see below). While this is more complicated (and may involve 5D-field processes) photons (at least) behave as if they were oppositely charged pairs, which cancel each other’s charge and opposite XD-momenta⁹ -hence self confined to 4D.

Beyond being unobservable, the quantum processes which produce the photons’ zero velocity and radiation pressure in XD, are presently hidden “underneath” the elegant-looking, schematic condition XD=constant [7]. This exemplifies the fundamental importance of neo-KK approach to try to replace even simple and reasonable schematic preconditions by physical processes. However, **when** oppositely charged particles annihilate into photons/other electrically natural particles (ENP), Wesson and Liu’s hypothesis [18] suffices. But, most-if-not-all the fireball’s photons’ were produced this way. Whether at the hadron era’s equilibrium reaction ($\gamma + \gamma \leftrightarrow \text{lepton} + \text{antilepton}$); at the lepton era’s muons’ disappearance ($\mu_+ \mu_- \rightarrow 2\gamma$); or even at the vast/total positrons’ disappearance ($e^+ + e^- \rightarrow 2\gamma$) by the excess of electrons (presumably) produced by the very early anti-proton decay. If valid, this 5D-process takes part in the universe’s earliest (KK) dimensional collapse: from massive and charged particles with **5D**-velocity (“5D-plasma”), to photons and other ENP with velocity in only 4D. This transforms the KK paradigm, from the mathematically possible¹⁰ “induced (4D) matter from empty 5D-GR”, to the physically predicted “produced (4D) ENP from (\pm)charged 5D-densities”.

⁹Photons might vibrate-but-not-propagate in XD, **as** they do in the ordinary two dimensions orthogonal to their propagation.

¹⁰The mathematical equivalence [4,6] alone does not demand any creation. As stressed by Wesson [e.g 3] even the 4D appearance might not mean creation, but rather how 4DU/5DU appears to 5DU/4DU-observers.

The “5D-plasma” may require 6D-empty-GR (see I, 3) or 5D (\pm) curvatures /fluctuations around zero [17]. This transformation will take us even further in chapter (V).

Notice that the fireball’s distribution is not predicted to be 4D. Only the ENP’s XD-velocity is paralyzed. This is why a neo-KKU **remains** 5D, yet bolometrically disconnected: photons do not illuminate all matter. By contrast – as gravitation is not XD-paralyzed, the XD-collapse processes continue, thereby narrowing the XD-distribution (but much more in IV).

Notice also that nD-KKU analysis of earlier eras might easily refute the entire neo-KK approach – or at least impose constraints on n – especially if found to disagree with the observed (and successfully predicted) cosmic abundance. The neo-KK approach hence owes to conduct this complicated examination regardless of our specific photonland hypothesis. Since we do not predict the virtual photons to have zero-XD velocity, such an examination will not help here (see also [24]).

(IV) The photonland hypothesis is intrinsically **compelled to predict** another XD-physical-process. Namely, if photons have no velocity in XD, a “radiation dominance” can never exist in XD(s)! Lacking radiation pressure in XD, even the hottest photonball cannot stop the matter’s gravitational collapse along XD(s), as it does in 4D. Thus “photonland” definitely predicts that the 5D-fireball’s matter has **already** collapsed in XD since its very appearance – “preparing ground” (mainly ENP densities, see below) for the galaxies’/clusters’ formation – along the **entire** radiation dominant era.

While this self-imposed, strange prediction of an “extended collapse” might easily complicate/refute the photonland hypothesis – it might no less than replace old-KKU’s artificial/hardly physical compactification mechanisms! Unlike the “cylindricity” abstraction, which is either unphysical, or hyperenergetic to an extent that generates radical disagreements with observations [e.g. 7], this natural gravitational collapse process neither requires a circular typology nor is required to “squeeze out” the XD(s) for us! With a mass-E-distribution along a not-at-all-negligible XD, our 4D-hypersurface is gradually produced – just as the galaxies/clusters are formed when - much later - photonland is freed from photon-dominance!

But moreover, (without trying to prematurely¹¹ develop a detailed pic-

¹¹These early XD(s) collapse processes might take various forms, and even invoke Alfvenic components.

ture) this 300,000 year period of undisturbed XD-collapse obviously mitigates – if not eliminates! – the harsh quandary of primordial density-fluctuations, which are presently **radically** insufficient for galaxy formation. A variety of KK inflationary scenaria is now expected to eliminate the insufficiency trouble; and a 5D one might suffice [17]. **And**, while this extreme insufficiency has hitherto compelled us to predict only a prebaryonic collapse - “photonland” (partially/totally) replaces it by it’s long pretransparent¹² DM-collapse! If [18] is valid, this pretransparent gravitational formation of our 4D-bubble is partially a dimensional collapse, since the 5D-free charged-pairs annihilate into the XD-paralyzed photons/ENP. This is crucial in determining whether/when the hypersurface’s XD-thickness (and our XD-blindness) ceases to be macroscopic, and remains significant “only” to quantum processes [24].

Finding “photonland” predicting/affecting/speculating fundamental cosmological processes, we must try to refute /prove it in all possible ways; especially by:

- 1) Quantifying at least the ($e^+ + e^- \rightarrow 2\gamma$) 5D-processes, and their resulting densities, pressures, temperatures, and mean free paths’ relationships to the universes’ radia.
- 2) Quantifying our radically extended pretransparent XD-collapse processes, and their variety of possible disagreements with standard cosmology, the present DM/matter ratio, and particle physics. Notice that if the dimensional collapse is shown **not** to be partial-this is a full compactification. “Photoland” ceases to explain the DM, and reduces to a compactification mechanism candidate.
- 3) Upgrading Wesson & Liu’s hypothesis to explain (and eventually predict) how the massless and chargeless photons must have zero XD-propagation, while the massless and chargeless gravitons do have XD-propagation, hence are clearly insensitive to XD. Please recall that gravitons are as admissible in neo-KKU as photons are [e.g. 5].

The photoland hypothesis is clearly contradictory to Gogberashvili’s new theory [19], which (apparently?) relies on photons’ non-zero velocity in five dimensions. He must therefore explain/predict (see above) how his theory does not destroy the conventional, successful predictions of both the radia-

¹²Thus, the pretransparent collapse, not the prebaryonic one, must eventually predict - and be tested against - the above DM/matter ratio.

tion and the matter dominated eras' processes/abundance, and the observed density and temperature of the present CMBR.

(V) Beyond predicting the gravitational-bolometric XD-gap, hence unmissing the (presently called) “missing matter” **and** the vast primordial densities-photonland is also our **observational prison** which is of an (annoying) fundamental importance to physics. In neo-KKU, 4D-GR is embedded in a flat hyperspace; its' 4D-densities interact with the 5D-solitons/ other “extended objects”; but our photon-dependent-observations are confined to 4DU. **This** is not merely “an unfortunate choice of coordinates” [7]. If the photonland hypothesis is valid, quantum processes wipe out our illusion¹³ of choosing observations beyond the photons' geometrical self-confinement! While this involves deeper philosophy of science which seems inappropriate here-choosing to add “photonless observations” of XDM thereby developing a higher dimensional perspective, is not unscientific/unreliable just because it transcends photonland. On the contrary, our photon-dependent-perspective is intrinsically biased; this has caused physics to miss most of the universe's matter, and to misperceive our 5DU as a narrow 4DU. If/as 5D-DM interacts with us (at least) gravitationally, XDM and 5D-densities are neither unreal, nor untrustable. Even under the extreme assumption that all real particles are 4D-bodies/fluctuations, and all 5D-DM are only virtual particles [24], their systematic analysis in a 5D frame of reference is “meta-optica”, not “metaphysica”. Hence, if our **present** universe is 5D, the possibilities of 5D massive/massless, charged/chargless, virtual particles (if not even quarks and leptons) must be vehemently scrutinized. Being XD-blind – unmissing the missing matter must also imply undismissing most of our universe's matter and volume as merely “dark”!

Both Wesson and Halevi¹⁴ predict our 4DU to be “a hypersurface in a flat 5D manifold” [3]. As long as it is **induced** by mathematical equiva-

¹³This inversely resembles the quantum processes which wipe out the illusion of certainty in observations: As photons cannot push-hence-distort our XD-observational-targets, we cannot observe them.

¹⁴In [17], extending “empty” 5D-GR to (E-non-violating) **fluctuations** of (\pm) curvatures/densities **around** zero, predicts a (hyperlow-probable phase- transitional) inflation-collapse process, where the 5D-negative densities hyperexplode into a 5D-hyperspace (thus integrating a physical, self-diminishing positive cosmological constant into the process) and the exploded positive densities collapse into the 4D- hypersurface/Einstein's spacetime (gradually **and** gracefully).

lences it might be “an effect of a choice of coordinates in a truncated higher-dimensional geometry” [7]. But, if it was physically produced by the pre-transparent dimensional collapse-these processes do not leave us (confused by) choices: They simultaneously show how both our hypersurface **and** our XD-blind observations/distorted frame of reference are formed! Thus, neither “the big bang is an artifact produced by an unfortunate choice of coordinates” [7], nor are our photon-dependent 4D-observations a (free) choice. We are just collectively afflicted by the “cylindricity-sight disorder”... Hence – if “photonland” is found valid – it also eliminates unclearities such as ...the Hubble expansion, the microwave background, and primordial nuclearsynthesis... are in a sense recognized as geometrical illusions – artifacts of a choice of coordinates in a higher-dimensional world” [5].

The photonland hypothesis must eventually refute or integrate another strange possibility: any 5D-neo-KKU is deep enough to enable $\binom{4}{3}$ – if not $\binom{5}{3}$ – orthogonal collapse processes into 4 groups of photons (and other real particles), each group with zero velocity in **another** dimension. Analyzing these primeval complicated 5D-plasma processes might easily refute – or gradually reshape/integrate the photonland and Wesson & Liu’s hypotheses. If photonland survives, the three “alien” photon groups – instead of illuminating our 4DU– might illuminate our understanding in 5D-quantum processes [24].

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